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New Estimate for the Time-Dependent Thermal Nucleosynthesis of ^{180m}Ta

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Abstract

We have made a new time-dependent calculation of the supernova production ratio of the long-lived isomeric state ^{180m}Ta. Such a time-dependent solution is crucial for understanding the production and survival of this isotope. We include the explicit linking between the isomer and all known excited states. We have also calculated the properties of possible links to a conjectured excited state which might decrease the final isomer residual ratio. We find that the explicit time evolution of the synthesis of ^{180}Ta using the available nuclear data avoids the overproduction relative to ^{138}La for a ν process neutrino temperature of 4 MeV.

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The nucleosynthesis of ^{180}Ta has remained an unsolved problem. For the most part this nucleus is bypassed by the major nucleosynthesis mechanisms of the s and r processes. This accounts for why this isotope is the rarest in Nature. For this reason, a variety of more exotic processes, such as a weak branch through excited states in ^{180}Hf [1], the β decay of ^{179}Hf followed by neutron capture [2], and the γ process [3–6], have been proposed and studied experimentally [7–9]. Perhaps, the most popular scenario in recent times is ^{180}Ta production in the ν process [10, 11] via the ^{181}Ta$(\nu, \nu' n)^{180}$Ta and ^{180}Hf(ν_e,e)^{180}Ta neutrino reactions in core-collapse supernovae. It is currently believed that only two isotopes (^{138}La and ^{180}Ta) among the heavy elements may be predominantly synthesized by the ν process [11]. Although the new calculated result based upon results of a recent nuclear experiment [12] can reproduce the Solar abundance of ^{138}La with charged current reactions and an electron neutrino temperature of 4 MeV, it overproduces the abundance of ^{180}Ta. Here we investigate the possibility that this overestimate originates from the unique feature that the naturally occurring abundance of ^{180}Ta is actually a meta-stable isomer (half-life of $\geq 10^{15}$ yr), while the true ground state is a 1^+ unstable state which β-decays with a half-life of only 8.15 hr (see Fig. 1). Therefore, a crucial ingredient for all of the possible production scenarios is the ratio of the population of the meta-stable isomer to the total production of this isotope.

In the ν process, low-spin excited states in ^{180}Ta are strongly populated from ^{180}Hf by Gamow-Teller transitions and subsequently decay preferentially to the 1^+ ground state [11] (a similar situation for ^{138}La is discussed in Ref. [13]). However, in a high temperature photon bath, the meta-stable isomer is excited from the ground state by (γ,γ') reactions through highly excited states. Moreover, the transition rate between the ground state and the isomer is affected by the changing temperature. Therefore, the final isomeric branching ratio should be evaluated by a time-dependent calculation.

Previous studies have noted [11, 12] that the observed ^{180m}Ta abundance can not be inferred from their calculations until the branching between the long-lived isomer and the ground state is known. If one knew sufficient information on transitions between the two states of ^{180}Ta, the branching ratio could be calculated by the mathematical method of [14]. However, although many experiments have been carried out to identify the paths between the two states in ^{180}Ta [15–19], such paths have never been identified and only transition probaiblities have

been measured [8, 20]. In this letter, therefore, we develop a new calculation method which can be applied even with limited information. This method is then applied to provide a realistic estimate of the branching in ^{180}Ta. We show that this model leads to a good agreement between the final calculated isomeric abundance and the observed Solar-system abundance for a broad range of astrophysical parameters.

We adopt an exponential decrease $T = T_0 \exp(-t/\tau)$ as a reasonable approximation to the adiabatic expansion of shock heated material in supernovae [3]. Nuclei are completely thermalized in the high temperature regime so that the population ratio of any two states is simply given by a quotient of their Boltzman factors, i.e. $m_i/m_j = (2J_i + 1)/(2J_j + 1)\exp[-(E_i - E_j)/kT]$, where m_i denotes the population of the state of i, with spin J_i, and excitation energy, E_i. For $T_9 = 0.1{-}1.0$ (where T_9 is the temperature in units of 10^9 K) all excited states lower than a few hundred keV are populated. After the freeze-out each excited state decays to either the ground state or the isomer and must be considered [22].

In the transitional region, strongly connected states are only partly thermalized. For our purposes we can model the excited-state structure of a deformed nucleus as consisting of two sets of nuclear states: 1) the ground state structure, which consists of the ground state plus the excited states with strong transitions to the ground state; and 2) the analogous isomeric structure. The transition probability between states of the two structures depends upon their quantum number K which is the projected component of the total angular momentum along the nuclear symmetry axis [23]. Transitions with $\Delta K > \Delta I$ are forbidden, where ΔI is the transition multipolarity. In the case of ^{180}Ta the ground state and the isomer have $K = 1$ and 9, respectively. Thus, the two structures can only communicate by weak linking transitions and each structure can be considered independently thermalized (see Fig, 2). We can therefore treat these structures as two independent nuclear species.

To construct a model for the time evolution we consider two simple cases of linking transitions as shown in Fig. 2. In the first case there is a single linking transition between states 2 and 3. In this case, the time-dependent evolution of the population probability of the ground-state structure, $N_0 = \sum m_i^g / (\sum m_i^g + \sum m_j^m)$, is given by

$$dN_0/dt = -P_2^g \rho B_{23} N_0 + P_3^m A_{32}(1 - N_0) \quad , \tag{1}$$

where A_{32} and B_{23} are the Einstein coefficients between the indicated states, ρ is the photon density of a thermal Plank distribution, and $P_i^{g(m)}$ is the normalized population ratio of the excited state, $P_i^{g(m)} = m_i^{g(m)}/\sum m_j^{g(m)}$. In the case of $kT << (E_2\text{-}E_1)$, the Einstein coefficients are related by $\rho B_{12} = (g_2/g_1)A_{21}\exp\left[-(E_2 - E_1)/kT\right]$, where $g_i = (2J_i+1)$ is the spin statistical factor, and we obtain,

$$dN_0/dt = -(g_3/g_0)\exp\left[-(E_3 - E_0)/kT\right]P_0^g A_{32}N_0 + (g_3/g_1)\exp\left[-(E_3 - E_1)/kT\right]P_1^m A_{32}(1-N_0)\ . \tag{2}$$

A similar expression exists for the case of a single linking transition between excited states 4 and 5 on Fig. 2. It is straightforward to extend this to the general case of multi-linking transitions. The time-dependence of the population probability of the ground state structure is then given by

$$\frac{dN_0}{dt} = -\sum_{i,p} P_i^g A_{ip} N_0 + \sum_{i,p} P_i^m \rho B_{pi}(1 - N_0), -\sum_{j,q} P_j^g \rho B_{qj} N_0 + \sum_{j,q} P_j^m A_{jq}(1 - N_0)$$

$$= -\sum_{i,p} P_0^g \frac{g_i}{g_0}\exp\left[-(E_i - E_0)/kT\right]A_{ip}N_0 + \sum_{j,q} P_1^m \frac{g_j}{g_1}\exp\left[-(E_j - E_1)/kT\right]A_{jq}(1 - N_0), \tag{3}$$

where, 0 and 1 denote the ground state and the isomer, respectively, i (j) denotes levels of the ground state structure (or the isomer structure) and p (q) denotes the levels of any other structure.

In general, excited states of deformed nuclei are characterized by collective rotational motion. Each excited state is a member of a rotational band, in which the electric transition probabilities are enhanced by 1−2 orders of magnitude. However the interband transition probabilities between two such rotational bands are much hindered relative to the intraband transitions. Therefore, in the case of deformed nuclei, the Γ_i corresponding to the interband transition rates are much smaller than Γ_0 and one can approximate $g_i/g_1\Gamma_i\Gamma_0/\Gamma \approx g_i/g_1\Gamma_i$. Finally, inserting $A = \Gamma/\hbar$, into equation (3), we obtain

$$\frac{dN_0}{dt} = -\sum_i P_0^g \frac{g_1}{g_0}\exp\left[-(E_i - E_0)/kT\right]\frac{g_i}{g_1}\frac{\Gamma_i}{\hbar}N_0 + \sum_j P_1^m \exp\left[-(E_i - E_1)/kT\right]\frac{g_j}{g_1}\frac{\Gamma_j}{\hbar}(1 - N_0). \tag{4}$$

We have applied this general formula [Eq. (4)] to the case of ^{180}Ta. The excitation energies and spins have been taken from an evaluated data set of Ref. [24]. We have calculated $P_i^{g(m)}$

by taking into account all known excited states up to 600 keV excitation energy. The isomer population ratio $P_1^m = m_1^m / \sum m_i^m$ is 0.85−0.94 during the transitional temperature region of T_9 = 0.44−0.62 (see below). This result indicates that the linking transitions connected directly with the isomer are crucial for the total transition rate between the ground state and isomeric structures. Note that the transition between states 2 and 3 in Fig. 2 are negligibly small since the spins of the excited states above the isomer are generally larger than the spin of the isomer ($J = 9$).

Belic *et al.* reported on the partial widths $g_i/g_1 \Gamma_i \Gamma_0/\Gamma$ [meV] [20] for 9 transitions connected with the isomer. However, transitions between the ground state and the excited states connected to the isomer have not been identified (see Fig. 1). Figure 3 shows the calculated isomer population ratio taking into account the 9 transitions in ^{180}Ta. For initial conditions we begin with static thermal equilibrium among states at $T_9 = 1.0$, and we take $\tau = 1$ s for the supernova temperature time constant. The population of the isomer structure decreases with decreasing temperature as expected. In the high temperature region ($T_9 > 0.62$) the present calculated result is identical with that obtained by assuming static thermal equilibrium. However, in the low temperature region ($T_9 < 0.62$) the present time-dependent calculated result is significantly different from the thermal equilibrium result. In the freeze-out region at low temperature ($T_9 < 0.44$) the two structures are completely disconnected and the isomer population ratio remains fixed at $P_m/(P_m+P_{gs}) = 0.39 \pm 0.01$. The uncertainty which is evaluated from the experimental errors of the energy width Γ_i [20] is small since both the transition rates of m $\rightarrow$ gs and gs $\rightarrow$ m are proportional to Γ_i.

For completeness we should also remark that it has been suggested [22] that an unobserved linking transition to a state at 592 keV might also exist. To estimate the transition probability, we plot in Fig. 4 the 9 known linking transition widths to the isomer as a function of their excitation energy. The transition widths increase with excitation energy and can be fit with a simple exponential growth. This trend can be understood in terms of a K mixing effect caused by the Coriolis interaction, but a detailed explanation of why a simple exponential growth reproduces the observed transition probability so well is beyond the scope of this Letter. Nevertheless, from a least-square fit to this trend (dashed line on Fig. 4) we can estimate that an unknown state at 592 (800) keV would have a width of $g_i/g_1 \Gamma_i = 0.003$ (0.008) [meV].

The isomeric residual population ratio is then $P_m/(P_g+P_m) = 0.18$ and 0.30 for hypothetical additional linking transitions to states at 592 keV and 800 keV, respectively. The isomer ratio is sensitive to the lowest energy of the linkning transitions. Note, however, that available experimental results for this nucleus have found no evidence for direct transitions below 1.09 MeV [20] or 0.739 MeV [25], and that previous γ-ray spectroscopy experiments [15–19] have measured no semi-direct transitions lower than 1.09 MeV; this fact indicates that the lowest energy of the linking transitions is the known energy of 1.09 MeV. We choose, therefore, $P_m/(P_g+P_m) = 0.39$ based only on the observed 9 linking transitions for the following discussion.

In the ν and γ processes, ^{180}Ta is initially synthesized in environments with $T_9 > 0.62$ so that ^{180}Ta is completely thermalized and the production ratio in any previous nuclear reaction process does not affect the final relative isomeric population. The isomer population at freeze-out is insensitive to the temperature time constant because of the slow rate of change of the population near the freeze-out. The calculated values are almost identical for temperature timescales in the range of $\tau = 0.3{-}3$ s. As we shall now show, the only remaining sensitive astrophysical parameter is the neutrino temperature for the ν process.

Byelikov et al. [12] presented nucleosynthesis yields and it was concluded that ^{138}La was reproduced by neutrino charged current reactions and ν_e temperatures around 4 MeV. However, ^{180}Ta was overproduced without correcting of the isomer residual ratio. In Table 1, we present the modified yields taking into account the isomer residual ratio. The quoted production ratios of ^{138}La and ^{180}Ta are normalized to ^{16}O which is known to be predominantly produced in supernovae and its solar abundance is very large compared with that of the heavy elements. A successful nucleosynthesis mechanism must produce abundances relative to ^{16}O near unity. Otherwise, these isotopes are under (or over) produced. Both ^{138}La and ^{180}Ta are now produced in relative amounts near unity by the charged current neutrino reactions and a ν_e temperature of 4 MeV. Although this result is only from the 15 $M_\odot$ model the core properties do not depend much upon progenitor mass and the initial mass function is weighted toward low-mass progenitors. Hence the 15 $M_\odot$ model should be a good representation of a proper average over supernova progenitors.

We stress that, in the present calculation, the isomer ratio is almost independent of the astrophysical parameters such as the peak temperature, the temperature time constant, the

TABLE I: Nucleosynthesis production factor relative to Solar (normalized to ^{16}O) for various γ or ν process conditions (column 1) based upon the 15 $M_\odot$ supernova model of Ref. [11]. In the notation of Ref. [12], + n.c. means the γ process plus neutral-current interactions, and ++c.c. denotes that charged current interactions are also included at the indicated neutrino temperatures. Column 2 gives the total ^{138}La yields and column 4 gives the total (g.s. + isomer) ^{180}Ta yields [11, 12]. The yields of ^{180m}Ta (column 3) deduced in the present work are based upon our calculated isomer residual population fraction of 0.39. Note that for $T_\nu = 4$ MeV ^{138}La and ^{180m}Ta are produced with about the same over-production factor.

Model	^{138}La	^{180m}**Ta**	^{180}Ta(g.s.+isomer)
γ-proc. only	0.190	**0.234**	0.599
+ n.c. (6 MeV)	0.280	**0.399**	1.024
++ c.c. (4 MeV)	1.101	**1.218**	3.123
++ c.c. (6 MeV)	2.797	**2.109**	5.409
++ c.c. (8 MeV)	3.222	**2.881**	7.387

supernova neutrino energy spectrum, and the explosion energy. The final result that the over-production problem of ^{180}Ta in the ν process is largely reduced is probably robust, though only a full set of stellar nucleosynthesis calculations that include the time-dependent depopulation of the isomer will answer this question quantitatively. As shown in Table I, the present result constrains the neutrino energy spectrum, which is of importance for understanding the supernova explosion mechanim and the detection of supernova neutrinos in near future. A previous study [26] has shown that light element yields of the ν process depend upon the neutrino oscillation parameter Θ_{13}. Heger *et al.* suggested that the yields of ^{138}La and ^{180}Ta is sensitive to Θ_{13} [11]. Using the present isomer ratio, one can now study systematically the neutrino oscillation effect for both of light and heavy elements.

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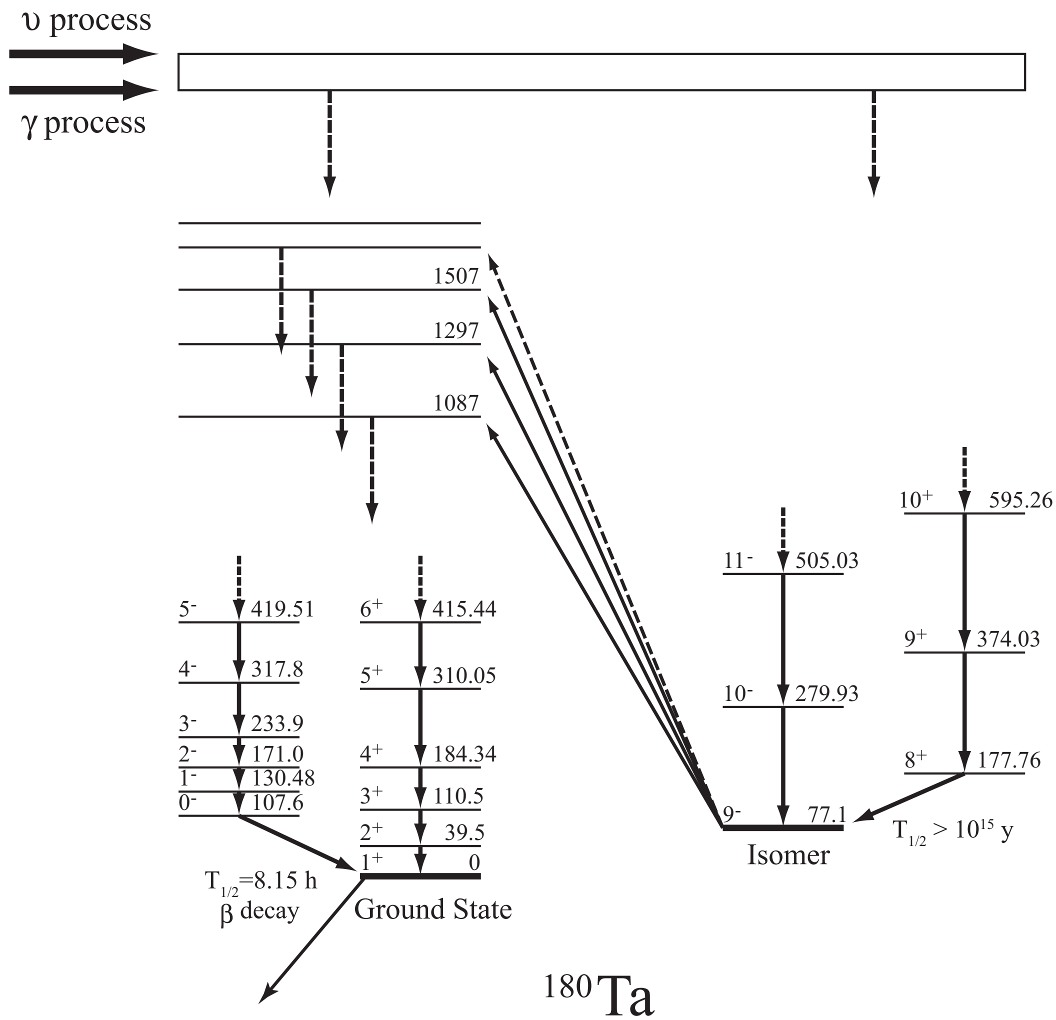

FIG. 1: Partial nuclear level scheme of ^{180}Ta. The ground state decays via β decay with a half-life of 8.15 h, while the isomer is a meta-stable state. The measured [20] excitation energies of states with weak transitions to the isomer are indicated. These excited states should also connect to the ground state but their decay paths are unknown.

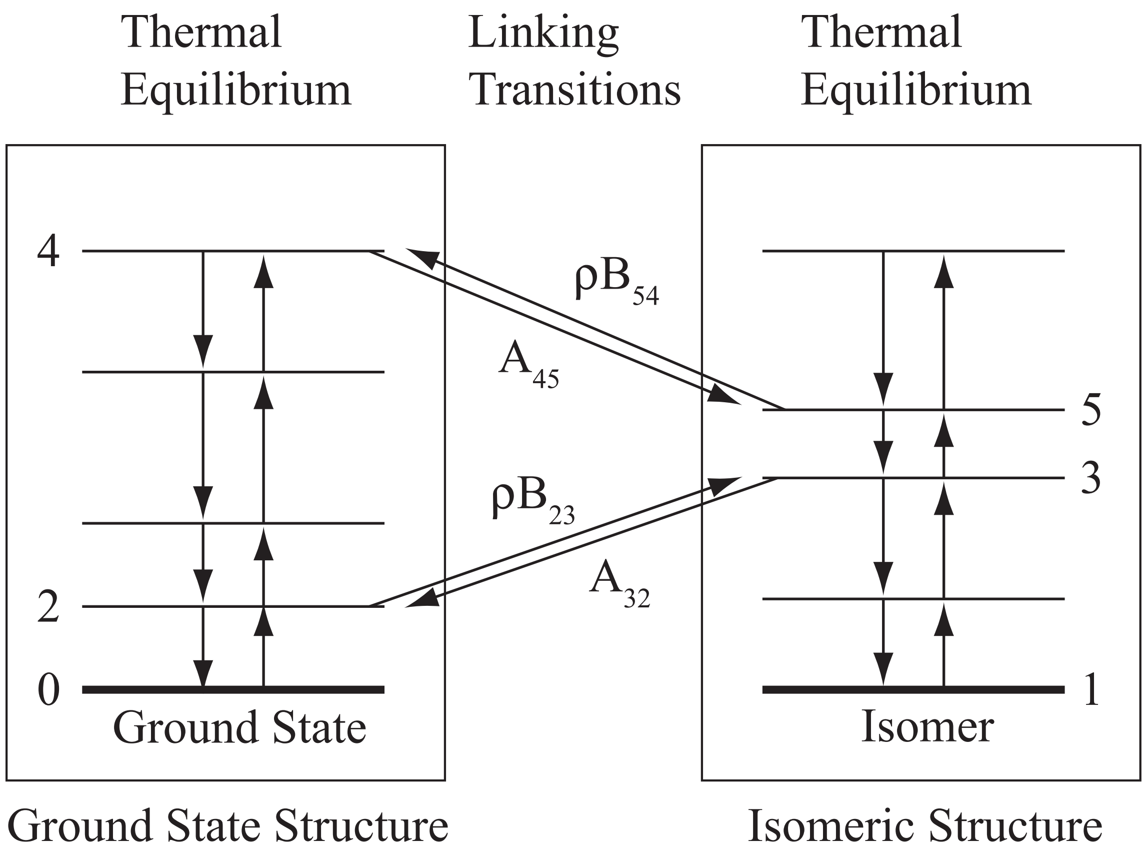

FIG. 2: Schematic illustration of the nuclear structure relevant during the transitional temperature region. The ground state structure (i.e. the ground state and excited states above the ground state) is in thermal equilibrium. The isomeric structure is also in thermal equilibrium. The ground state and isomeric structures are connected via the indicated linking transitions.

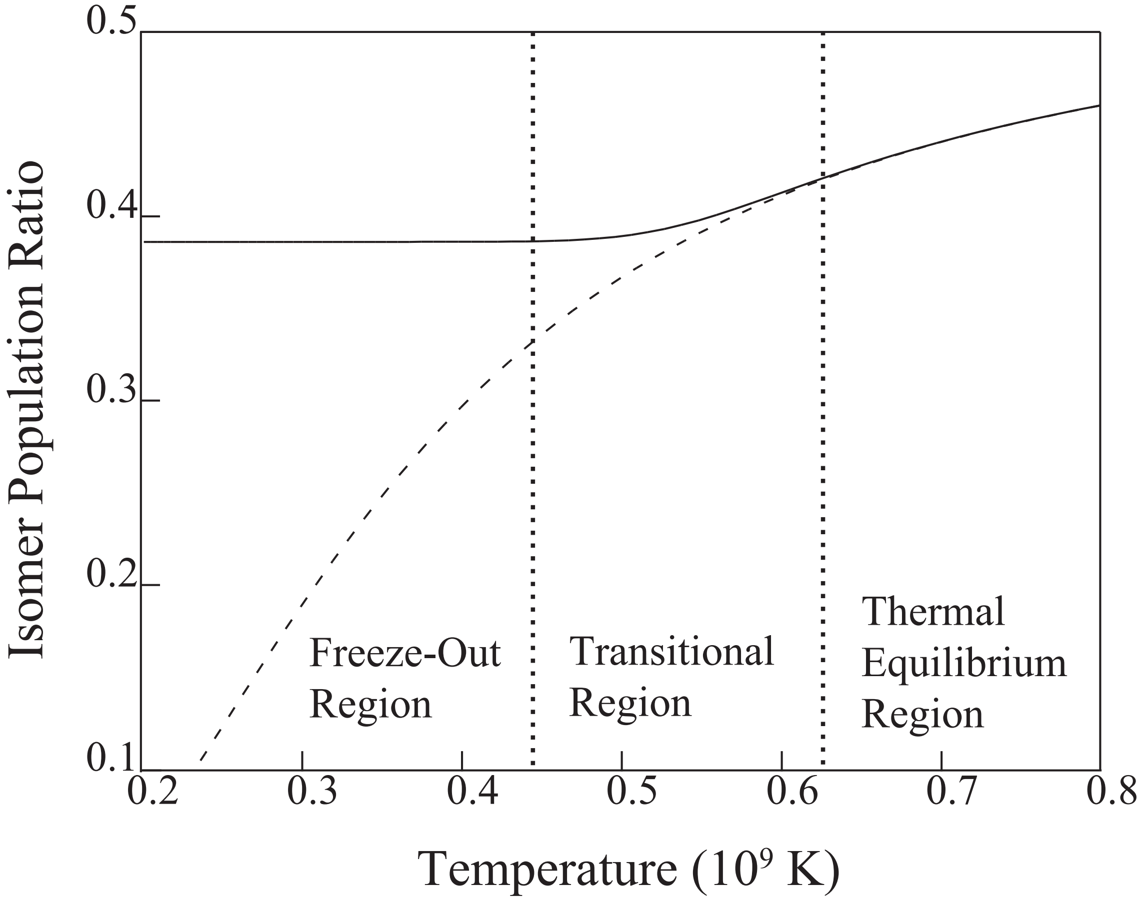

FIG. 3: Calculated isomer population ratio. The solid line denotes the time-dependent calculated result. The dashed line denotes the isomer ratio under the condition of thermal equilibrium. In the high temperature regime ($T_9 > 0.62$) both are identical. In the low temperature regime ($T_9 < 0.62$) the calculated ratio is much higher than the thermal equilibrium ratio.

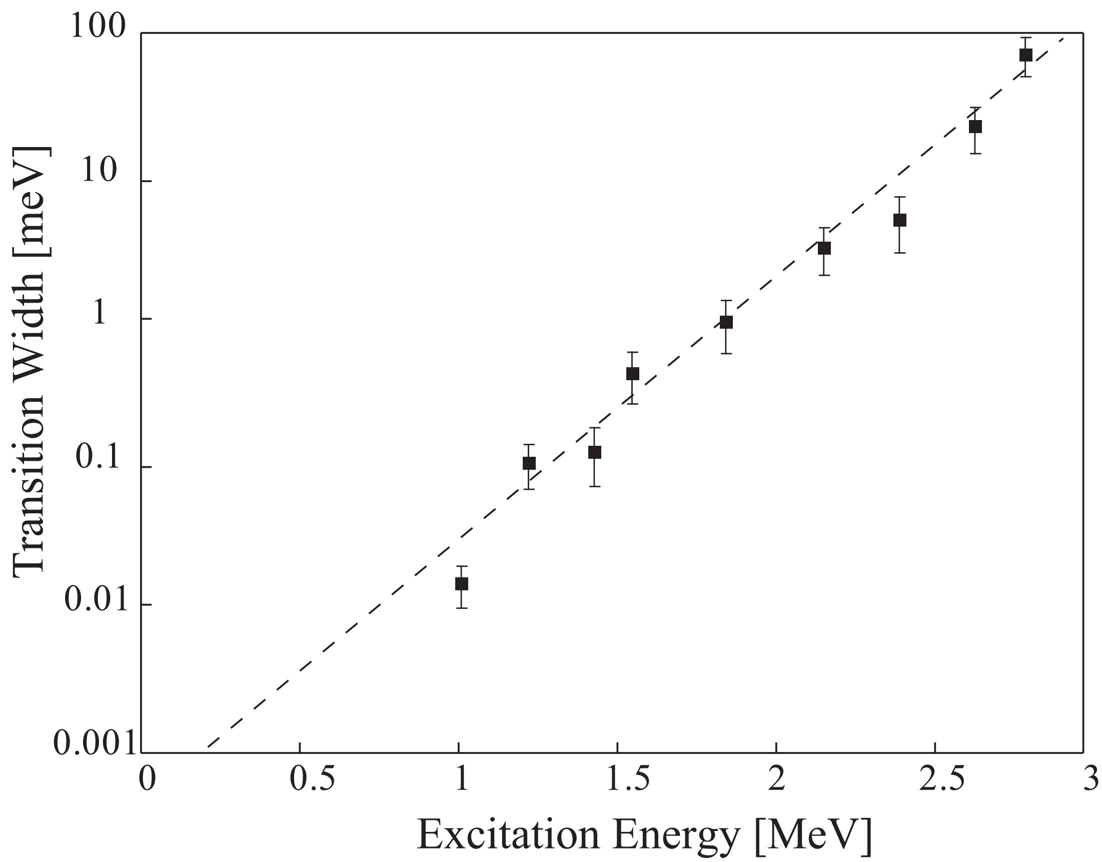

FIG. 4: Observed transition widths $g_i/g_1\Gamma_i\Gamma_0/\Gamma$ [meV] of the 9 known linking transitions as a function of their excitation energy. The Data are taken from Ref. [20]. The dashed line is a simple exponential least square fit to these widths used to estimate widths for other hypothetical linking transitions.